

Regular Article

An Adaptive Bandwidth Notch Filter for GNSS Narrowband Interference Mitigation

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Abstract– The low level of received signals power makes Global Navigation Satellite Systems (GNSS) receivers vulnerable to many classes of disturbing signals. Among them, narrow band interference (NBI) might cause serious receiver performance degradation. Cancellation of NBI can be implemented by using notch filters (NF), which are controlled by two parameters: the notch frequency, which specifies the band center, and the notch bandwidth, which defines the spectrum area to be removed. The literature on the topic focuses on adapting the notch frequency, without estimating the filter bandwidth. This paper proposes a method able to determine both the notch parameters, optimizing the interference suppression. The performance analysis shows a good improvement by using the adaptive bandwidth notch filter.

Keywords– Satellite Navigation, GNSS, Radio Frequency Interference (RFI), Adaptive Bandwidth Notch Filter, Interference Mitigation.

1 INTRODUCTION

Though the main applications based on the satellite navigation are related to the transports domains, nowadays the Global Navigation Satellite Systems (GNSSs) play a fundamental role in several fields, belonging to very different areas. The tracking of vehicles fleets (trains, trucks, or vessels), the synchronization of communications and energy distribution networks, the logistics, the mapping of the territory and the systems for the coordination of the rescue teams are examples of important civil applications. Within this scenario, the requirements that the GNSS receiver has to fulfill, both in terms of accuracy and continuity, are becoming more and more stringent.

GNSS systems like GPS and Galileo are based on the Direct Sequence-Spread Spectrum (DS-SS) technique, which intrinsically gives them a high level of robustness [1]. At the same time, it has to be considered that the satellite signals arrive at the receiver antenna with an extremely low level of power (approximately 20 dB below the noise floor). This makes them vulnerable to different types of disturbance, among which the Radio Frequency Interference (RFI) from external sources is one of the main threats.

All the systems transmitting at carrier frequencies close to the band of interest are potential sources of interference for a GNSS receiver, and even small leakages out of their allocated bandwidth can become a threat for the receiver. Even though events of unintentional RFI are generally unpredictable, several cases have been experienced in the past and reported in literature [2–6].

Moreover, the growing number of wireless communication infrastructures increases the probability that some out of band energy affects the performance of GNSS receivers.

Normally, the RF interference is classified as wide-band (WB) or narrow-band (NB), depending on the ratio between the interference and the desired GNSS signal bandwidths [7]. In this sense, an interference might be classified differently, depending on the specific GNSS signal that is considered (e.g., the same interference can be classified as wide for the GPS L1 C/A code and narrow for the GPS P(Y) code). The limit for a narrow-band interference is a single tone [7], usually referred to as continuous wave (CW).

In the literature, several examples of proposed solutions able to detect (and eventually mitigate) interfering sources can be found [5, 8–10]. Normally, the monitoring techniques are properly designed to cope with a specific class of disturbances (wideband, narrowband, pulsed signals, etc.). In this paper we focus on the class of narrow-band interferers (NBI), where the term “narrow” has to be considered with respect to the GPS L1 C/A code and the Galileo E1 signals (i.e., the interference bandwidth is within tens of kilohertz).

Considering such a classification and in accordance with [11], for this kind of disturbing signals, the notch filter (NF) has proved to be an efficient mitigation technique, since it can be considered a good compromise between efficiency and complexity. When using a NF to remove narrowband interference, it has to consider its two parameters: notch frequency and notch bandwidth. The notch frequency decides the center frequency to

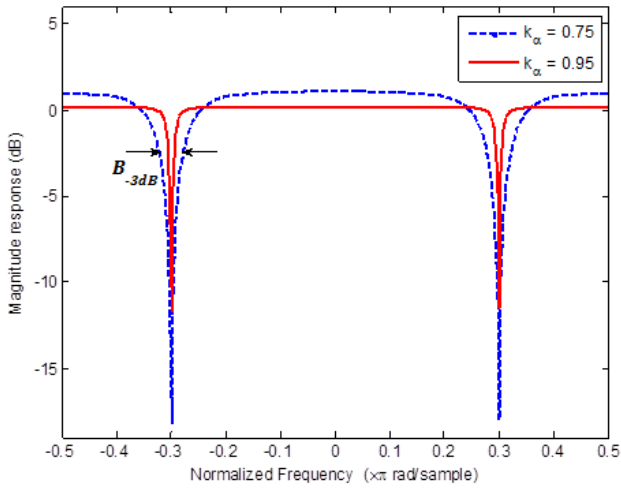


Figure 1. Frequency response of notch filter for two difference pole factors.

be removed. The notch bandwidth is the width of frequency region in which the signal power is reduced by 3 dB. The notch frequency can be adapted and track the interference frequency by using the Least mean square (LMS) algorithm [11]. However, the problem of determining the notch bandwidth has not been considered. To further improve the interference mitigation capability, in this paper, we propose an interference mitigation module based on adaptive notch filtering, which takes into account the adaptation of both the notch frequency and the notch bandwidth. The performance analysis proves an improvement of 1 to 3 dB for the proposed method with respect to the conventional one in the literature.

As for the paper structure, after this introduction, Section 2 introduces the application of the notch filter in the narrowband interference mitigation. Section 3 proposes two added methods in the interference mitigation module of a GNSS receiver: one for interference bandwidth estimation and the other for the NF configuration. Section 4 gives performance analysis of the proposed module, and Section 5 concludes the paper.

2 NOTCH FILTER FOR NARROWBAND INTERFERENCE MITIGATION

2.1 Overview of Notch Filter

As defined in [12], a notch filter (NF) is a filter that passes all frequencies except those in a stop or rejection band centered on a central frequency. Therefore, a notch filter can be defined by two parameters: the notch bandwidth B_N , and notch frequency f_N .

There are several ways to implement a notch filter, normally divided in three main classes: the Fast Fourier Transform (FFT)-based, the Finite Impulse Response (FIR), and the Infinite Impulse Response (IIR) notch filters [12]. In recent years, IIR notch filters are considered for GNSS interference mitigation because of their low computational requirements, efficient implementation and low number of parameters to be adapted [11].

The IIR two-pole NF has the following transfer function [11]:

$$H(z) = \frac{1 - 2\text{Re}\{z_0\}z^{-1} + |z_0|z^{-2}}{1 - 2k_\alpha\text{Re}\{z_0\}z^{-1} + |z_0|k_\alpha^2z^{-2}} \quad (1)$$

where $z_0 = re^{j2\pi f_N}$ is the zero, which corresponds to the notch frequency f_N , and k_α (with $0 < k_\alpha < 1$) is the pole contraction factor, which decides the notch bandwidth. To effectively exploit the capability of NF for removing interference signals, the zeros should be constrained to be on the unit circle [13].

Figure 1 reports the frequency responses of a notch filter with two different pole factors. It is clear that the pole factor k_α decides the width and the depth of the notch: the more the value of k_α is close to unity, the narrower and shallower the notch is. In other words, the notch bandwidth is characterized by the pole factor k_α , as it decides which frequency components and how much their power will be removed. Hence, the problem of determining k_α is very important.

2.2 GNSS Receiver Structure with NF Interference Mitigation Module

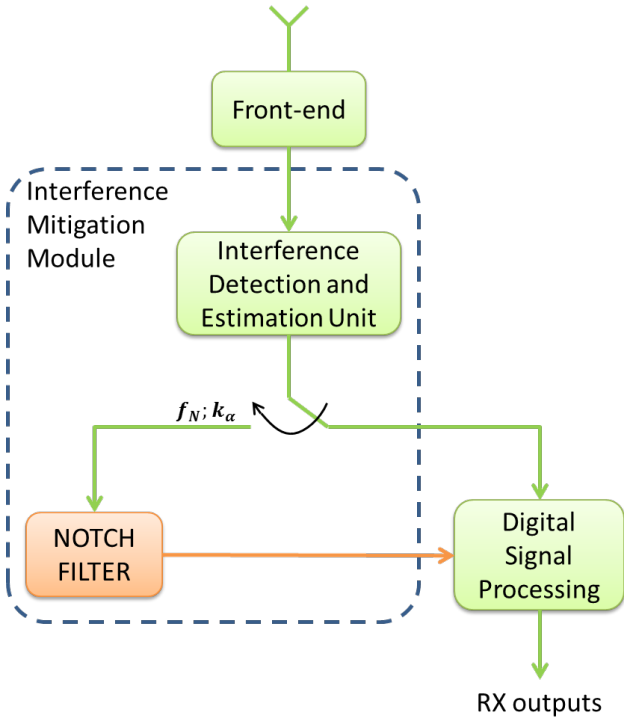
As mentioned in Section 1, the notch filter has been proved to be an efficient interference mitigation technique. Figure 2(a) shows a high-level block diagram of the GNSS receiver chain, where the Interference detection/estimation block and the Notch filter block are present. After arriving at the receiver's antenna, the GNSS signal (with the noise and the interference components) is filtered, down-converted to the Intermediate Frequency (IF), and sampled by the front-end. Then, the 'Interference Detection and Estimation unit' is in charge of (see Figure 2(b)):

- Deciding if the interference is present (and the notch filter needs to be activated);
- The interference frequency f_{int} and interference bandwidth B_{int} are estimated as \hat{f}_{int} and \hat{B}_{int} . Then, the NF parameters f_N and k_α are calculated based on the estimated values.

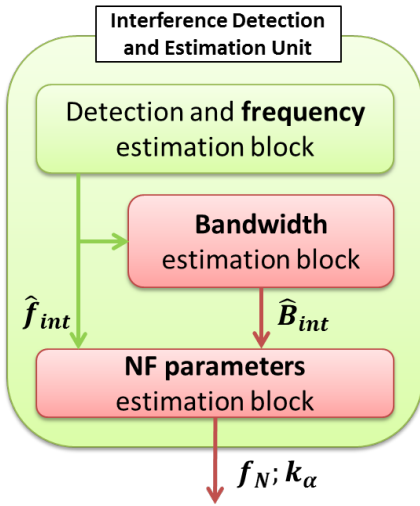
If no interference is detected, then the regular signal processing is implemented; otherwise \hat{f}_{int} and \hat{B}_{int} provided by the "Interference Detection and Estimation Unit" block are used to configure the NF, which is responsible for suppressing the interference components in the IF signal samples.

In reality, the characteristics of the interference signal vary with time. As an example, Figure 3 (from [4]) shows some plots of spectra evaluated (every 18 seconds over a 1 minute of data set) from live signals, collected in a hostile environment. It is evident how the signal spectrum changes significantly over time and several spurious peaks appear always over the same frequency ranges (i.e. [1-3] MHz and [6-8] MHz), but with different widths of band. For this reason, the notch filters are often designed in order to adapt and track the interference signal. The adaptation of the NF towards the interference signal variation along time is performed following the two steps below:

- Step 1: Continuously estimate \hat{f}_{int} and \hat{B}_{int} .



(a) High-level block diagram of the GNSS receiver chain



(b) Detection/Estimation Unit

Figure 2. High-level block diagram of the GNSS receiver chain.

- Step 2: Reconfigure the NF via resetting f_N and k_α based on the estimated values of f_{int} and B_{int} .

2.2.1 Interference Frequency Estimation (or Interference Detection):

When using NF to remove interference, it is necessary to know the notch frequency (or interference frequency). However, in reality, this parameter is unknown and might vary over time. Thus, an adaptive notch filter (ANF) is designed to estimate and track it automatically. To do this, it is important to note that if the notch frequency matches the interference carrier ($f_N = f_{int}$), the amount of removed power will be maximum, and the power of signal after filtering will be minimum. Due to this statement, the ANF can be implemented by using the Least Mean Square (LMS)

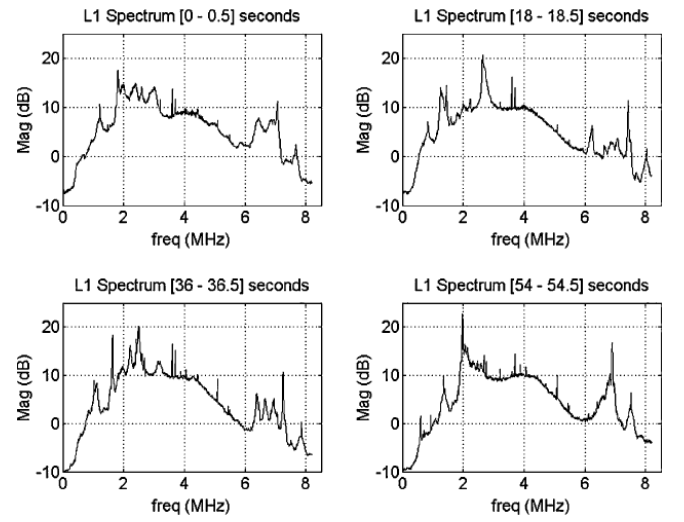


Figure 3. Spectrum at the front-end output at different time instants (from [4]).

algorithm to minimize the power of filter output, as follows [11]. The zero z_0 of transfer function (1) is updated iteratively. If the interference is present, the angle of z_0 will converge to the interference frequency. The iterative rule to update z_0 is

$$z_0[n+1] = z_0[n] - \mu \nabla_{z_0}(|y[n]|^2), \quad (2)$$

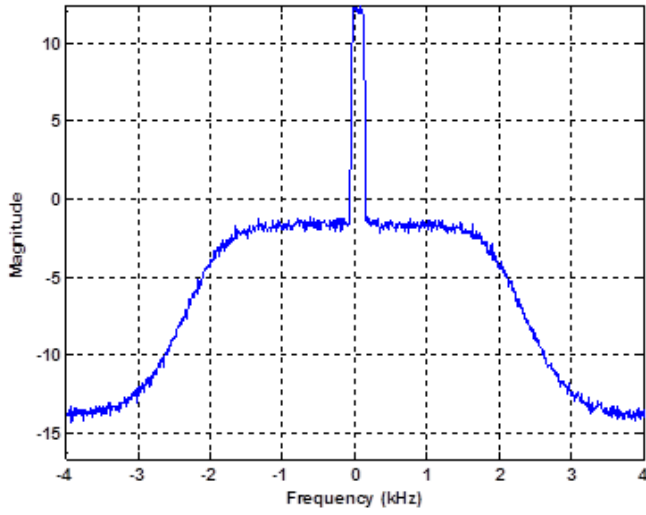
where ∇ denotes the stochastic gradient, μ is the algorithm step and $y[n]$ is the output of the notch filter [11]. The estimated interference frequency f_{int} can be calculated from the mean of z_0 after convergence.

Since z_0 is complex, the adaptation is performed with both the real and image part of z_0 . The simulation results showed that if any interference is present in the received signal, the adapted zero converges close to the unit circle. Therefore, [11] introduced a detection unit using the mean value of the amplitude of z_0 to check if the notch filter is tracking an interference.

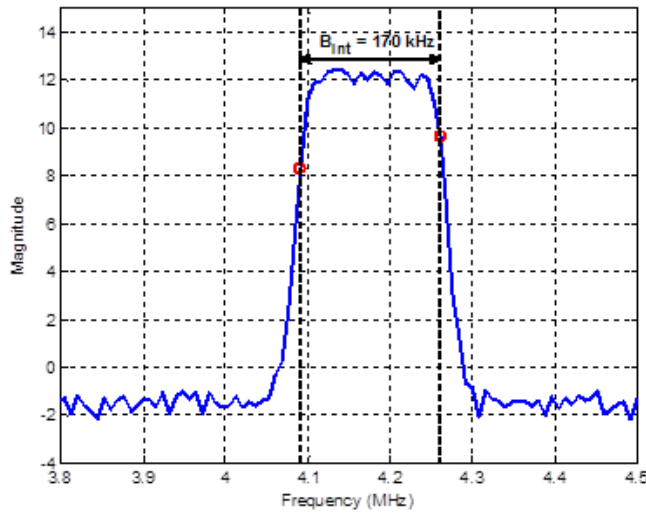
This type of filter can be used for both interference detection and mitigation [11]. However, since its zeros are not constrained to unit, this might reduce the capability of the NF for removing interference. Thus, it is only used for the Detection and frequency estimation block in this paper.

2.2.2 Interference Bandwidth Estimation:

Besides frequency, bandwidth is also an important characteristic of interference because it relates to the configuration of NF. As explained in Section 2.1, k_α decides the width of the removed frequency region. Thus, to determine k_α , the estimation of the interference bandwidth B_{int} is necessary. Furthermore, at the same time, not only the interference frequency but also the interference bandwidth can be variable as shown on Figure 3. This illustrates the need for tracking the change of the interference band. However, to the best of our knowledge, there has been no work on the estimation of B_{int} . Therefore, in Section 3 we propose two methods: one for estimating and tracking B_{int} and the other for estimating k_α for the NF from the estimated value.



(a) Spectrum of interfered signal



(b) Zoom in the interference band

Figure 4. Spectrum of the input signal.

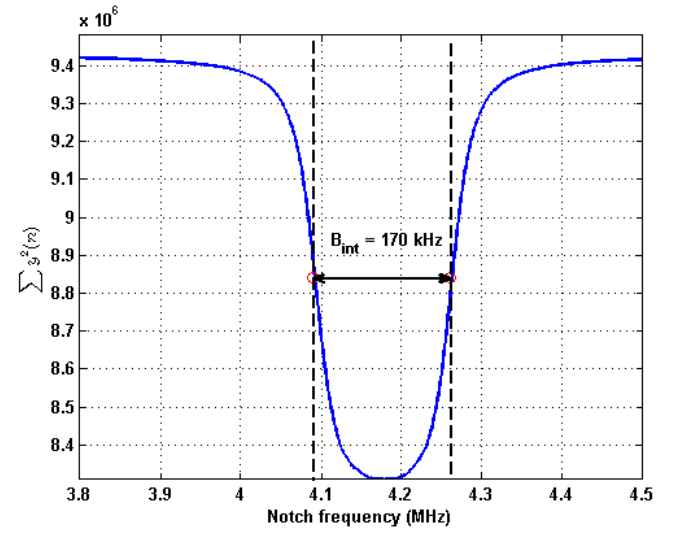
3 PROPOSED ADAPTIVE BANDWIDTH NOTCH FILTER

In this section, we introduce the methods to estimate B_{int} and choose k_{α} for the final NF. They are implemented in the Bandwidth estimation block and NF parameters estimation block in Figure 2(b).

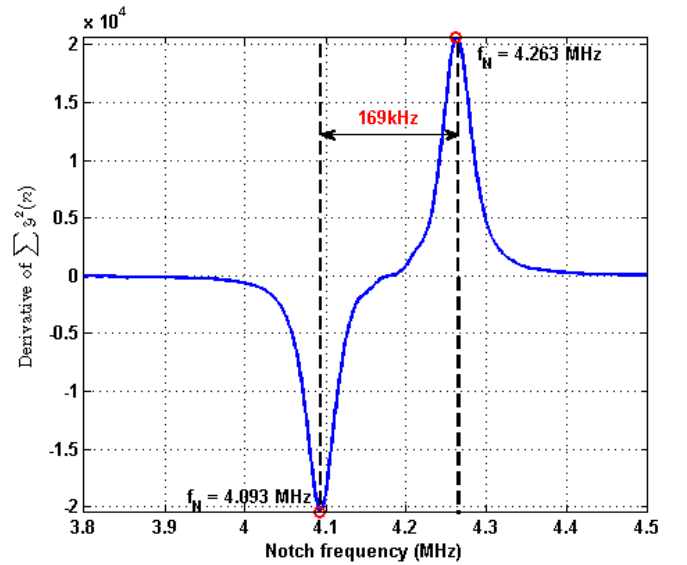
3.1 Interference Bandwidth Estimation

Firstly, consider a GPS signal generated by N-FUELS [14]. It contains a GPS PRN 1 signal in L1 band, at nominal outdoor signal power: 45 dB-Hz of carrier to noise ratio. It is affected by an interference signal with 170 kHz of bandwidth and frequency at 42 kHz from the GPS carrier (the worst interference frequency for PRN 1 [7]). The signal spectrum is shown in Figure 5(a). The x-axis is normalized with respect to the L1 carrier frequency ($f = 0$ corresponds to the GPS band center).

Figure 5(a) shows the signal power at the output of a NF, whose notch bandwidth is fixed at 30 kHz and whose frequency sweeps over the spectrum range.



(a) Signal power at the output of a NF



(b) Derivative of the signal power at the output of a NF

Figure 5. The signal power at the output of a NF and its derivative.

The two circles indicate the portion of the spectrum occupied by the interference ($B_{\text{int}}=170$ kHz). Furthermore, Figure 5(b) shows the derivative of the output power (in Figure 5(a)). It is easy to observe how the distance between the two critical points (maximum and minimum) is a good estimation of the interference bandwidth.

On the basis of the described approach, the proposed solution for the bandwidth estimation block follows the next procedure:

- Two notch filters with fixed predefined bandwidth are used with the scope of estimating the interference bandwidth B_{int} . Each NF works on a single portion of the spectrum (i.e., one NF finds the minimum and one NF finds the maximum of derivative of the filter output power), using the interference frequency estimation f_{int} as the starting point.
- The frequencies at the output of the two notch filters are the limits of the interference band.

This block is presented more clearly in Figure 6.

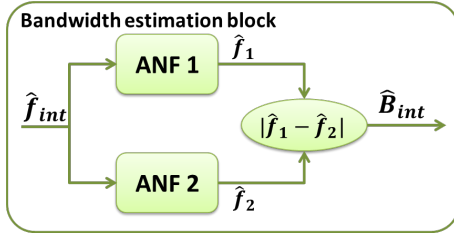


Figure 6. Bandwidth estimation block.

ANF 1 and ANF 2 can be implemented by the same way of the ANF in Section 2.2.1. However, the LMS algorithm is applied to minimize and maximize the derivative of the filter output power. Thus, instead of converging to the interference frequency, they will converge to the two limit frequencies of interference band. To exploit the capacity of interference removal, the zeros of NF in the paper are constrained to unit. Thus, the iterative rule for the updating process in (2) is considered only for the real part of zeros as follows:

$$\begin{aligned}\beta[n+1] &= \beta[n] \pm \mu \nabla_{\beta}^2(y^2[n]) \\ &= \beta[n] \pm \mu [(\nabla_{\beta} y[n])^2 + y[n] * \nabla_{\beta}^2 y[n]],\end{aligned}\quad (3)$$

where μ is the algorithm step [11], and $\beta(1) = \cos(2\pi f_{\text{int}})$. The operation ‘−’ is used for the ANF 1, which performs the maximization, and the operation ‘+’ is used for the ANF 2, which performs the minimization.

To reduce the number of variables needed to be adapted, the zeros of transfer function (1) are constrained to be on the unit circle. Thus, $\nabla_{\beta} y[n]$ and $\nabla_{\beta}^2 y[n]$ can be deduced from $y[n]$ as follows:

$$y[n] = 2\beta[n]k_{\alpha}y[n-1] - k_{\alpha}^2y[n-2] + x[n] - 2\beta[n]x[n-1] + x[n-2], \quad (4)$$

$$\begin{aligned}\nabla_{\beta}(y[n]) &= 2k_{\alpha}y[n-1] + 2k_{\alpha}\beta\nabla_{\beta}(y[n-1]) \\ &\quad - k_{\alpha}^2\nabla_{\beta}(y[n-2]) - 2x[n-1],\end{aligned}\quad (5)$$

$$\begin{aligned}\nabla_{\beta}^2(y[n]) &= 4k_{\alpha}\nabla_{\beta}(y[n-1]) + 2k_{\alpha}\beta\nabla_{\beta}^2(y[n-1]) \\ &\quad - k_{\alpha}^2\nabla_{\beta}^2(y[n-2]).\end{aligned}\quad (6)$$

When the LMS algorithm reaches its steady-state, the standard deviation of the output frequency has to be smaller than a threshold T . Moreover, during a convergence period the frequency deviation does not exceed the quarter of the filter bandwidth [15]. Thus, the stop convergence of ANF 1 and ANF 2 can be created when:

$$\begin{cases} \sigma(\cos^{-1}(\beta[n])) \leq T \\ |\cos^{-1}(\beta[D(k)]) - \cos^{-1}(\beta[D(k-1)])| \leq \frac{B_N}{4} \end{cases}$$

where $k \in N$, B_N is the bandwidth of filter, $\sigma(\cos^{-1}(\beta[n]))$ is the standard deviation of the output frequency during the period of D-iteration. For the simulations in the rest of this paper, $D = 5 \times 10^5$ and $T = 500$ to ensure both the speed and the accuracy of the stop of convergence.

Consequently, the bandwidth of interference can be calculated from the mean values at their output frequency during their convergence period, as follows:

$$\hat{B}_{\text{int}} = |\hat{f}_1 - \hat{f}_2| = \frac{|\cos^{-1}(\bar{\beta}_{\text{ANF1}}) - \cos^{-1}(\bar{\beta}_{\text{ANF2}})|}{2\pi} \quad (7)$$

where the bar symbol denotes the mean value.

3.2 Adaptive Bandwidth Notch Filter Configuration

As point out in the Appendix, the carrier to noise ratio C/N_0 of the received signal after filtering can be expressed as

$$\frac{C}{N_0} = \frac{\frac{C_s}{N_0} \left| \int_0^{\frac{F_s}{2}} H(f) S(f) df \right|^2}{\int_0^{\frac{F_s}{2}} |H(f)|^2 S(f) df + \frac{C_i}{N_0} \int_{f_{\text{int}} - \frac{B_{\text{int}}}{2}}^{f_{\text{int}} + \frac{B_{\text{int}}}{2}} |H(f)|^2 S(f) df} \quad (8)$$

where F_s is the sampling frequency, $H(f)$ is the transfer function of filter, $S_i(f)$ and $S(f)$ are the normalized power spectral density of the interference and the GNSS signal, C_s, C_i are the power of the received GNSS signal and the interference signal, respectively, B_{int} is the bandwidth of interference and f_{int} is the interference frequency.

Figure 7 shows the theoretical line of C/N_0 with a typical signal available to the commercial GPS receiver with interference bandwidth $B_{\text{int}} = 25$ kHz and $P_i/P_s = 30$ dB (data demodulation threshold for the wide-band interference) [16]. The interference frequency matches the GPS signal frequency. Since the large k_{α} leads to the distortion of the useful signal, k_{α} in the range $[0.9, 1)$ is often chosen in experiments [11, 13, 17]. [17] recommended that k_{α} should be set at 0.9. In this experiment, the notch filter is investigated with k_{α} larger than 0.9. It can be seen that the best k_{α} is about 0.97.

According to [17], the relation between k_{α} and the notch bandwidth can be estimated as follows:

$$B_N = \frac{F_s(1 - k_{\alpha})}{\pi}. \quad (9)$$

Thus, k_{α} of 0.97 corresponds to the notch bandwidth of about 155 kHz ($F_s = 16.3676$ MHz). This value does not match the interference bandwidth, because C/N_0 depends on not only the interference bandwidth, but also the power of interference. Thus, it is not possible to use the estimated interference bandwidth to calculate k_{α} directly by (9). In this paper, k_{α} will be estimated based on the expression (8).

It has to note that there are four parameters to be considered in (8): (i) the power of the useful signal P_s , (ii) the noise density N_0 , (iii) the interference bandwidth B_{int} and frequency f_{int} , and (iv) the power of interference P_i . However, P_s does not change the shape of the C/N_0 line, so it is not necessary to know this value in advance. N_0 can be calculated from the temperature of the environment. f_{int} and B_{int} are provided by the Detection and estimation unit. and P_i can be estimated by spectrum analysis and refined after signal processing. In this section, these parameters are assumed to be

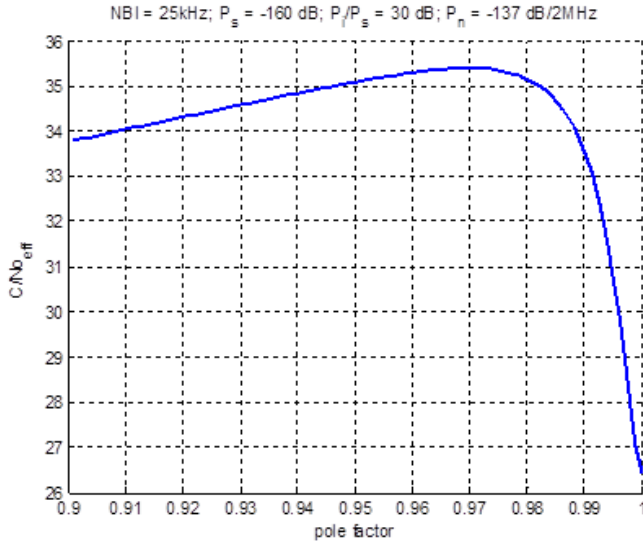


Figure 7. Theoretical C/N_0 when bandwidth of interference $B_{int} = 25$ kHz (ACARS Harmonics) and $P_i/P_s = 30$ dB.

known in advance. Now, (8) has only one variable k_α . However, it is difficult to find the value of k_α directly from

$$\hat{k}_\alpha = \arg \max_{k_\alpha} \left\{ \frac{C}{N_0}(k_\alpha) \right\} \quad (10)$$

Fortunately, since the pseudo-range code of GNSS signal has the line spectrum spaced at 1 kHz, Equation (8) can be discretized and calculated fast and easily with varying k_α . Thus, the problem (10) can be solved approximately by the numerical method. Then, the configuration of NF can be completed.

4 PERFORMANCE ANALYSIS

4.1 Bandwidth Estimation

In this section, the simulation results obtained by implementing the bandwidth estimation block will be presented. The characteristics of the input signal are as introduced in Section 3.1. The notch bandwidth of the two ANFs is fixed at 30 kHz. The estimated frequencies at their output are shown in Figure 8. The two black lines correspond to the actual limits of the interference bandwidth, while the red and blue lines are their estimation.

Figure 9 shows the achieved result with the simulated signal, which has the configuration as in Section 3.2. Similar to Figure 9, ANF 1 and ANF 2 work well on estimating the interference bandwidth.

The stop of convergence is also shown in Figures 8 and 9. In the case of a wide narrow-band interference ($B_{int} = 170$ kHz) as in Figure 8, the algorithm is found to reach the steady state after 2.5×10^6 iterations (about 0.153 s). For the narrower interference band ($B_{int} = 25$ kHz) in Figure 9, the time requirement is shorter with 10^6 iterations (about 0.06 s). The bandwidth of some unintentional interference sources for L1 band is under 100 kHz. For example, the Aircraft Communication Addressing and Reporting System (ACARS) with 25 kHz,

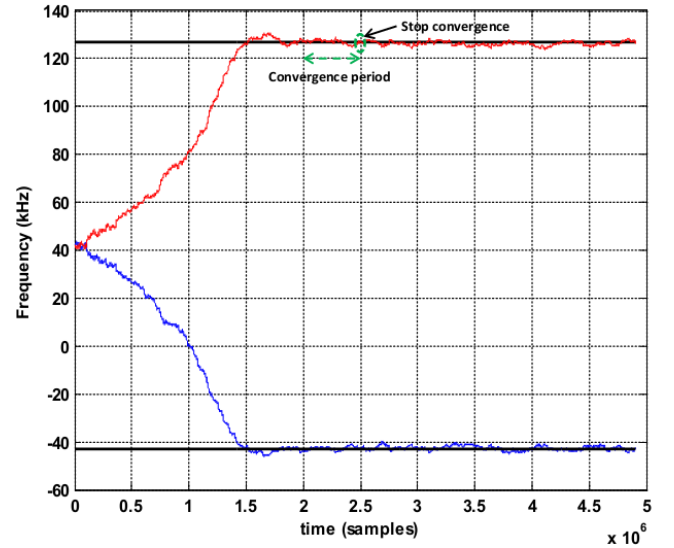


Figure 8. Simulation results: estimation of the interference bandwidth ($B_{int} = 170$ kHz).

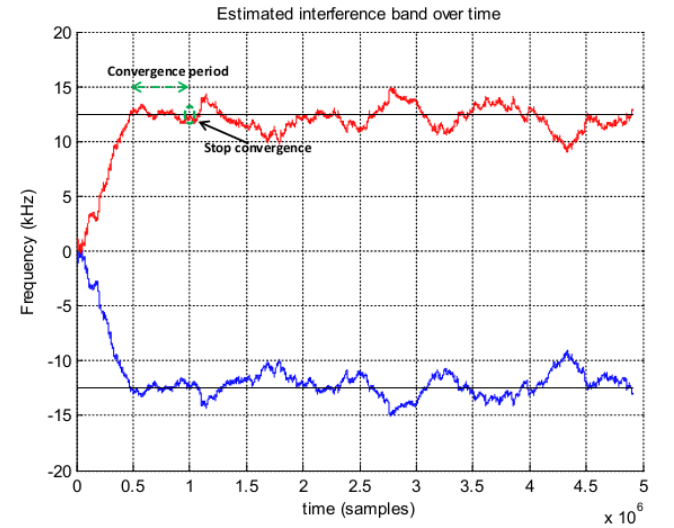


Figure 9. Simulation results: estimation of the interference bandwidth ($B_{int} = 25$ kHz).

and the SATCOM Communications with 20 kHz [16]. Thus, the rate of convergence as the results is acceptable.

Furthermore, the performance of the algorithm has also been tested in case of variable the interference bandwidth. Figure 10 shows the results when the algorithm is applied to a data set of 1 s containing an interference source, whose characteristics vary over time (i.e., the interference bandwidth is 10 kHz over the first 500 ms, and 18 kHz over the last 500 ms of simulation).

Although the proposed bandwidth estimation block is able to estimate interference band, it has a limit related to the selection of prefixed bandwidth of ANF 1 and ANF 2. It can be seen in Figure 11, which shows the results of the interference bandwidth estimation for different choices of NF bandwidth. The red line indicates the real interference bandwidth, i.e., $B_{int} = 170$ kHz. It is worth noticing how the algorithm is able to finely

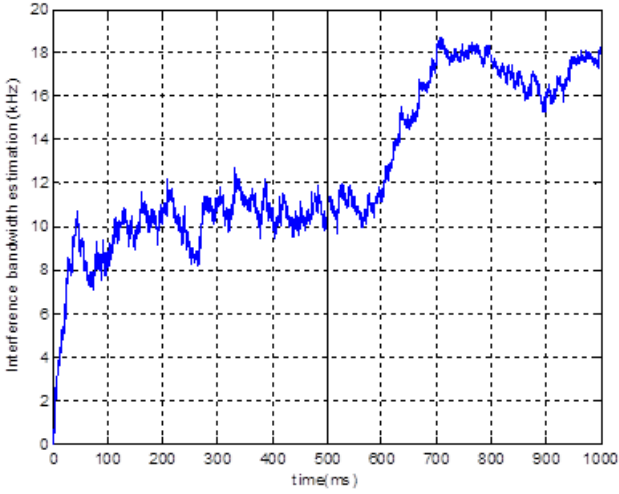


Figure 10. Estimated interference bandwidth over time, in case of variable band.

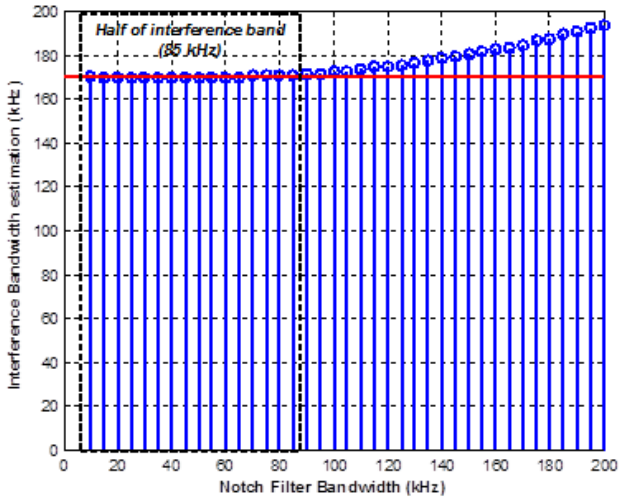


Figure 11. Interference bandwidth estimation for different NFs bandwidth.

estimate the bandwidth, when the width of the NFs bands is less than half of B_{int} .

4.2 Notch Filter Parameters Adaptation

In [17], the authors recommended that k_α should be set at 0.9 to provide good performance, whereas, in [13], $k_\alpha = 0.99$ is recommended. In this section, a comparison the quality of the signal after filtering by NFs with three different k_α is given, where two are recommended in [17] and [13], and the other is calculated by the proposed method in Section 3. The sequence number of codes and Doppler shift frequency of the satellites presented in the input signal are chosen randomly. The interference frequency matches the intermediate frequency: $f_{\text{int}} = f_{\text{IF}}$. The used method to estimate C/N_0 is Signal to Noise Variance [18].

It can be seen from the Table I that a too narrow notch does not remove enough the interference power; then it might lead to the signal loss of track. Whereas, a too wide notch leads to the loss of the useful signal, then reduces the value of C/N_0 . In the presence of the

proposed interference mitigation module, the quality of the signal is improved more than 1 dB compared to the cases of $k_\alpha = 0.9$ and $k_\alpha = 0.99$. This is because the removed power of the interference is balanced with the unwanted loss of the useful signal in presence of the proposed module. Therefore, a module to tune this factor, as proposed in Section 3, is necessary.

5 CONCLUSION

Narrow-band interferers can be classified among the most dangerous threats for GNSS receivers. For this kind of disturbing signals notch filtering has proved to be an efficient mitigation technique, because it can be considered a good compromise between efficiency and complexity.

The use of NF in interference mitigation requires the knowledge of interference characteristics. Much research in the past only focused on determining the interference frequency. In the current work, we proposed a technique that aims at estimating and tracking the bandwidth of the filter. Then, an expression of C/N_0 was given to evaluate the quality of the filtered signal. It is also used to optimize the configuration of NF.

Simulation results showed that the proposed methods can estimate the bandwidth of interference and choose an appropriate pole factor for NF. Though some open issues still have to be solved, the promising outcomes give the motivation for further investigations. Future work will mainly focus on the tuning of the bandwidth of the two notch filters used for the interference characterization.

APPENDIX A

PROOF OF EQUATION (8)

The received signal can be expressed by

$$s(t) = g(t) + w(t), \quad (11)$$

where $g(t)$ is GNSS signal; $w(t)$ is noise, which includes white noise and interference.

Assume that the baseband signal is processed by a correlator, with the integration time T and local code $m(t)$, the correlator output of each component is:

$$R_s(0) = \int_{-\frac{T}{2}}^{\frac{T}{2}} g(t) m(t) dt, \quad (12)$$

$$R_w(\tau) = \int_{-\frac{T}{2}}^{\frac{T}{2}} w(t) m(t - \tau) dt. \quad (13)$$

According to Plancherel theorem [19], correlation of the useful signal can be written as

$$\begin{aligned} R_s(0) &= \int_{-\frac{T}{2}}^{\frac{T}{2}} g(t) m(t) dt \\ &= \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} X(f) \overline{M(f)} df, \end{aligned} \quad (14)$$

Table I
C/N₀ OF GPS SIGNALS AFTER FILTERING BY NF. THE INTERFERENCE BANDWIDTH $B_{int} = 25$ kHz

P_i	PRN	Doppler shift	$k_\alpha = 0.9$	$k_\alpha = 0.99$	Proposed Method	
					C/N ₀	$\hat{B}_{int}, k_\alpha^{optimize}$
−126.5 dB	1	2500	38.90	34.79	38.85	$\hat{B}_{int} = 23.6$ kHz, $k_\alpha = 0.965$
	5	1600	39.26	35.23	39.41	
	7	5000	38.68	Loss of track	38.59	
	12	4200	38.93	35.23	38.95	
−135.5 dB	1	2500	40.16	40.82	41.66	$\hat{B}_{int} = 25.1$ kHz, $k_\alpha = 0.98$
	5	1600	39.85	41.29	41.91	
	7	5000	39.85	40.97	41.61	
	12	4200	40.19	41.25	42.07	

where F_s is the sampling frequency, $X(f)$ and $M(f)$ are Fourier transforms of $g(t)$ and $m(t)$, respectively. $M(f)$ denotes the complex conjugation.

If the received signal goes through a filter with transfer function $H(f)$, (14) can be rewritten as

$$R_s(0) = \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} H(f) X(f) \overline{M(f)} df, \quad (15)$$

where C_s is the post-correlation power of GNSS signal, $S(f)$ denotes normalized power spectrum of the GNSS signal. Since $g(t)$ is modulated by the pseudo-range code, the power of useful signal pre-correlation is

$$\begin{aligned} P_s &= |R_s(0)|^2 \\ &= 2C_s \left| \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} H(f) |M(f)|^2 df \right|^2 \\ &= 2C_s \left| \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} H(f) S(f) df \right|^2 l. \end{aligned} \quad (16)$$

For noises, pre-correlation power of noises can be expressed

$$P_n = \frac{1}{T} \int_{-T/2}^{T/2} (R(\tau))^2 d\tau. \quad (17)$$

According to Parseval's theorem [19] and cross-correlation theorem:

$$\begin{aligned} P_w &= \frac{1}{T} \int_{-T/2}^{T/2} (R(\tau))^2 d\tau \\ &= \frac{1}{T} \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} |\mathcal{F}(R(\tau))|^2 df \\ &= \frac{1}{T} \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} |H(f) W(f) M(f)|^2 df. \end{aligned} \quad (18)$$

It is possible to rewrite $|W(f)|^2$ as $N_0 + C_i S_i(f)$, where N_0 is the white noise power density, C_i is the interference power and $S_i(f)$ is the normalized power spectrum of interference.

From (16) and (18), pre-correlation signal to noise

ratio is:

$$SNR =$$

$$\frac{\frac{2TC_s}{N_0} \left| \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} H(f) S(f) df \right|^2}{\int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} |H(f)|^2 S(f) df + \frac{C_i}{N_0} \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} |H(f)|^2 S(f) S_i(f) df} \quad (19)$$

According to [18], the C/N₀ ratio can be calculated as:

$$\frac{C}{N_0} = \frac{SNR}{T}, \quad (20)$$

where T is the correlator's integration time.

Thus, applying (19) and (20), we have

$$\frac{C}{N_0} = \frac{\frac{2C_s}{N_0} \left| \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} H(f) S(f) df \right|^2}{\int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} |H(f)|^2 S(f) df + \frac{C_i}{N_0} \int_{-\frac{F_s}{2}}^{\frac{F_s}{2}} |H(f)|^2 S(f) S_i(f) df} \quad (21)$$

Assume that the interference spectrum is concentrated in the range $[f_{int} - \frac{B_{int}}{2}; f_{int} + \frac{B_{int}}{2}]$, where B_{int} is the bandwidth of interference and f_{int} is the interference frequency. Since the signal spectrum is symmetric, (21) can be rewritten as:

$$\frac{C}{N_0} = \frac{\frac{C_s}{N_0} \left| \int_0^{\frac{F_s}{2}} H(f) S(f) df \right|^2}{\int_0^{\frac{F_s}{2}} |H(f)|^2 S(f) df + \frac{C_i}{N_0} \int_{f_{int} - \frac{B_{int}}{2}}^{f_{int} + \frac{B_{int}}{2}} |H(f)|^2 S(f) df} \quad (22)$$

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